Simple Cyclic loading model based on Modified Cam Clay

Implemented in CRISP main program version 2002.2 and higher
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Introduction

This report presents a simple soil model which provides a reasonable prediction of behaviour of clays under repeated loading. The model is based on the work of Carter, Booker and Wroth[1]. The model has been implemented into CRISP FE code. The parameters used are the same as in the existing Modified Cam Clay with the addition of one more parameter which characterizes cyclic behaviour. This parameter may be determined by performing cyclic triaxial tests under undrained conditions.

The Modified Cam Clay model

This model is based on Critical State Soil Mechanics. The main variables in CSSM theory are The effective mean stress p', the deviatoric stress q and the voids ratio e. These are defined as follows

$$p' = \frac{1}{3}\left(\sigma_1' + \sigma_2' + \sigma_3'\right) \qquad \textbf{Eq. 1}$$

$$q = \sqrt{\left\{\frac{1}{2}\left[\left(\sigma_1' - \sigma_2'\right)^2 + \left(\sigma_2' - \sigma_3'\right)^2 + \left(\sigma_3' - \sigma_1'\right)^2\right]\right\}} \qquad \textbf{Eq. 2}$$

$$dv = -\frac{de}{1+e} \qquad \textbf{Eq. 3}$$

where de is the change in voids ratio, and dv is the incremental volume strain

The Modified Cam Clay (MCC) model requires the specification of five parameters, values of which can be obtained from standard oedometer or triaxial compression tests. These parameters are:

- λ the gradient of the normal consolidation line in e-ln(p') space
- κ the gradient of the swelling and recompression lines in e-ln(p') space
- e_{cs} the critical voids ration which locates the consolidation lines in e-ln(p') space. This is taken as the voids ratio at unit p' on the critical state line
- M the value of the stress ratio q/p' at critical state. This is related to the angle of friction though the relationship $M = \frac{6\sin(\phi')}{3-\sin(\phi')}$
- G the elastic shear modulus

When a stress state is elastic, the following relationship is used:

$$\begin{Bmatrix} dp' \\ dq \end{Bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} dv \\ d\varepsilon \end{Bmatrix} \qquad \textbf{Eq. 4}$$

where K is the bulk modulus defines as

[1] Carter, J.P, Booker, J.R., Wroth, C.P. "A Critical State Soil Model for Cyclic Loading" published in *Soil Mechanics – Transient and Cyclic Loads*, 1982, John Wiley & Sons Ltd

$$K = \frac{(1+e)\,p'}{k}$$ **Eq. 5**

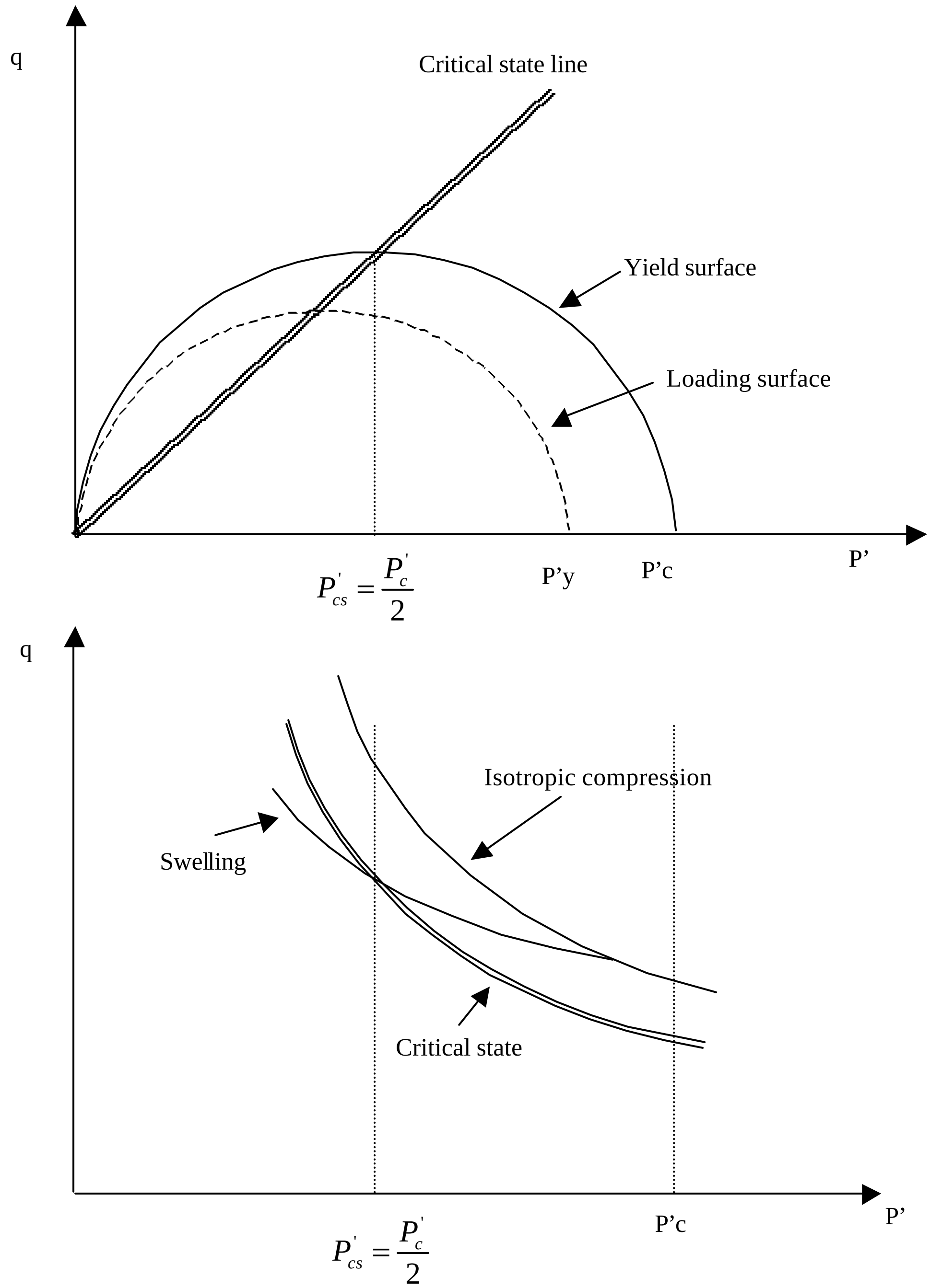


Figure 1 The Modified Cam Clay model

The function for the yield surface of the MCC model is defined as

$$q^2 - M^2\left\{p'(p'_C - p')\right\} = 0$$ **Eq. 6**

P'c is the preconsolidation stress which acts as a hardening parameter.

When a stress states satisfies the yield surface function, plastic deformation takes place, which is governed through an associated flow rule. The permanent change in incremental volumetric strain is given as

$$dv^p = \left(\frac{\boldsymbol{l} - \boldsymbol{k}}{1+e} \right) \frac{dp_c^{'}}{p_c^{'}}$$ **Eq. 7**

Each stress state has a loading surface, which is defined as:

$$p_y^{'} = p^{'} + \left(\frac{q}{M} \right)^2 \frac{1}{p^{'}}$$ **Eq. 8**

The following conditions apply when a material is loaded:

Hardening when $dp_y^{'} = dp_c^{'} > 0$

Softening when $dp_y^{'} = dp_c^{'} < 0$

Neutral loading when the yield locus does not change while plastic behaviour occurs, $dp_y^{'} = dp_c^{'} = 0$

During plastic behaviour the yield locus changes according to the law:

$$\frac{dp_c^{'}}{p_c^{'}} = \frac{dp_y^{'}}{p_y^{'}}$$ **Eq. 9**

The incremental stress-strain law during yielding is defines as

$$\begin{Bmatrix} dv \\ d\boldsymbol{e} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} dp^{'} \\ dq \end{Bmatrix}$$ **Eq. 10**

$$C_{11} = \frac{(\boldsymbol{l} - \boldsymbol{k})}{(1+e)} \frac{a}{p^{'}} + \left(\frac{\boldsymbol{k}}{1+e} \right) \frac{1}{p^{'}}$$

$$C_{12} = C_{21} = \frac{(\boldsymbol{l} - \boldsymbol{k})}{(1+e)} \left(\frac{1-a}{p^{'}} \right)$$

$$C_{22} = \frac{(\boldsymbol{l} - \boldsymbol{k})}{(1+e)} \left(\frac{b}{p^{'}} \right) + \frac{1}{3G}$$

$$a = \frac{M^2 - \boldsymbol{h}^2}{M^4 + \boldsymbol{h}^4}$$

$$b = \frac{4\boldsymbol{h}^4}{M^4 - \boldsymbol{h}^4}$$

$\boldsymbol{h}$ =the stress ratio q/p’

Cyclic Loading Model

Repeated loading usually causes the permanent strain to occur earlier than previous loading cycles. This implies that the yield stress limit is decreasing. In the conventional Modified Cam Clay model this cannot be reproduced as the yield surface is unaffected by activity in the elastic zone when the material is unloaded. In the new model it is assumed that the size of the yield surface reduces gradually with elastic unloading.

The following relationship is assumed when the loading surface $p_y^{'}$ is within the yield surface $p_c^{'}$ and is reducing (ie elastic unloading is taking place)

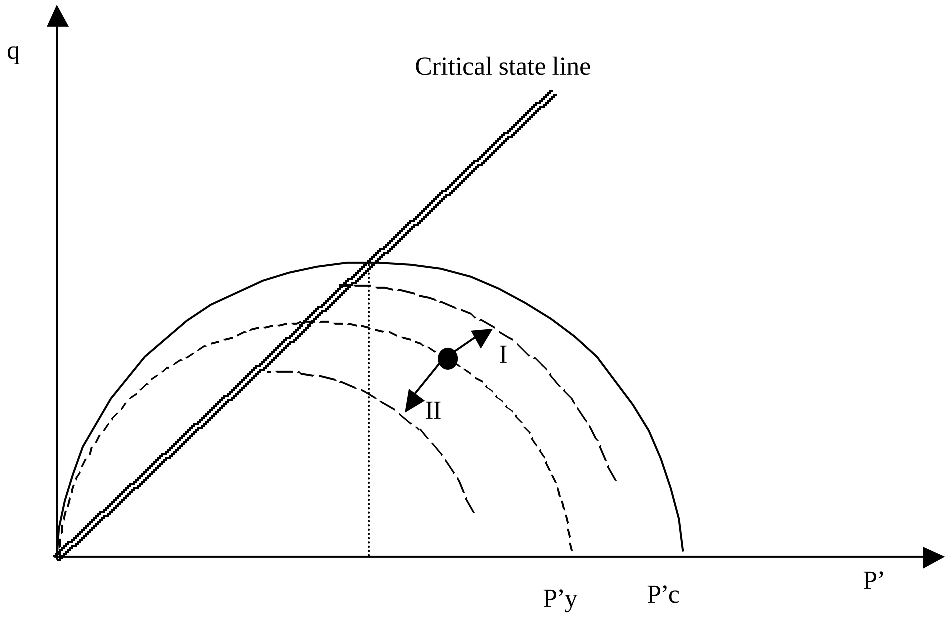
$$\frac{dp_c^{'}}{p_c^{'}} = J \frac{dp_y^{'}}{p_y^{'}} \qquad \textbf{Eq. 11}$$

If J is zero, then the model behaves exactly in the same was as the conventional MCC. If J is unity then the yield surface would contract so that the stress state remains on it. It is expected that the yield surface will contract only a small amount in which case J is a small value.

When the material is elastic and $p_y^{'}$ is increasing (ie elastic loading), then the current yield surface remains unchanged, ie

$$\frac{dp_c^{'}}{p_c^{'}} = 0 \qquad \textbf{Eq. 12}$$

The figure below shows the distinction between these two types of behaviour.



Path I represents elastic loading in which case $p_c^{'}$ is kept constant

Path II represent elastic unloading in which case $p_c^{'}$ contracts.

Figure 2 The new Cyclic Loading model

The new model is very similar to the conventional MCC. The criterion for yielding is the same, the flow rule and the hardening law are the same, and the incremental elastic and elasto-plastic stress-strain relations are the same. The only difference is the modification to the yield surface associated with elastic unloading. This feature has important consequences to the repeated loading problem, which would be useful for undrained as well as drained conditions.

Validation Examples on Normally Consolidated Clay

The following examples will be carried out using the following parameters:

$\lambda = 0.25 \quad \kappa = 0.25 \quad M = 1.2 \text{ for } \phi = 30^o \quad G = 200C_{uo}$

where C_{uo} is the initial undrained shear strength defined as

$$C_{uo} = \frac{M}{4} p'_{co} \left(\frac{2p'_o}{p'_{co}} \right)^{k/l} \qquad \textbf{Eq. 13}$$

The initial voids ratio is taken as e_o=0. This gives a critical voids ratio of:

$$e_{cs} = e_o + \kappa \ln(p'_o) + (\lambda - \kappa)\ln(p'_{cso}) \qquad \textbf{Eq. 14}$$

Model preparations

This example represents a triaxial test with cyclic axial load at constant cell pressure. In each case loading is applied so that the deviator stress is varied continuously between zero and a specified limit, ie one way compression where $\sigma_z \geq \sigma_r$ with σ_r constant.

We use ϑ=0.1 and $\frac{q_c}{2C_{uo}} = 0.75$

The FE model consists of 2 LST axi-symmetric elements, which will represent one quadrant of the triaxial sample due to symmetry. An initial stress of 150Kpa is applied to the sides as shown in order to satisfy equilibrium. The sample is assumed to be weightless (ie $\gamma_{bulk} = 0$). The initial fixities are as shown below.

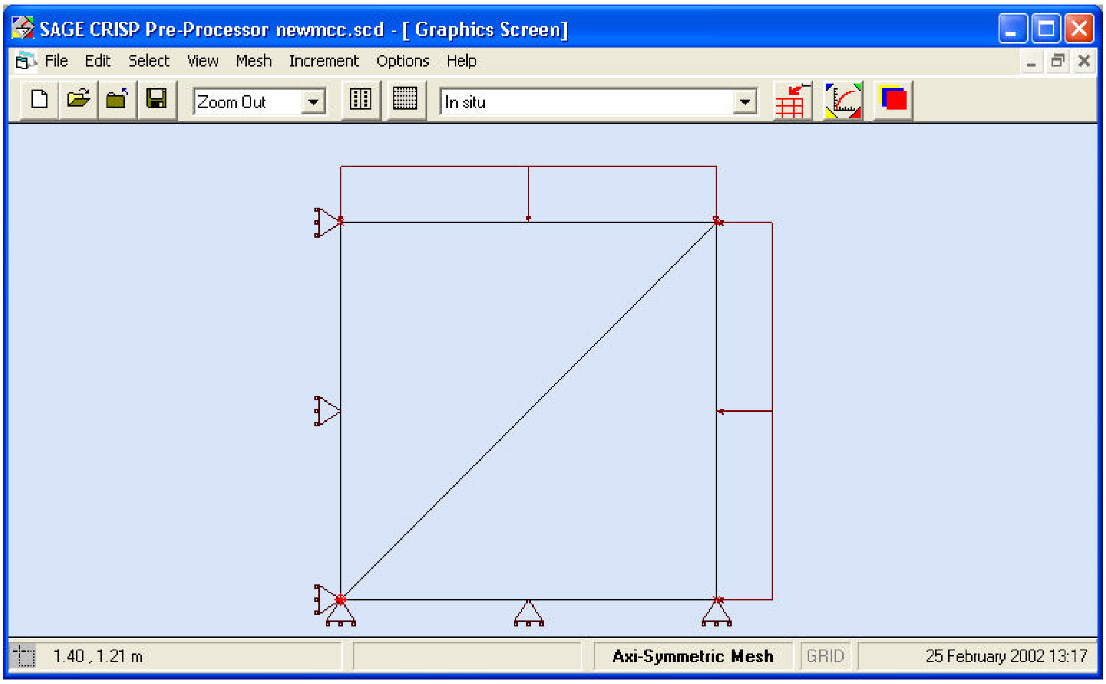


Figure 3 Starting mesh in SAGE CRISP

The in-situ stress is set as 150Kpa as shown below. As this is an undrained normally consolidated sample, $p'_{co} = p'_o = 150KPa$

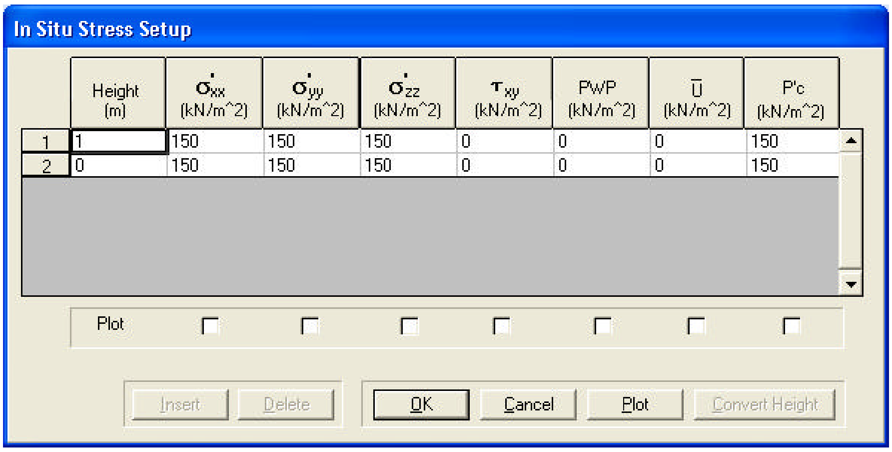


Figure 4 In-situ stresses

The material properties are entered as shown

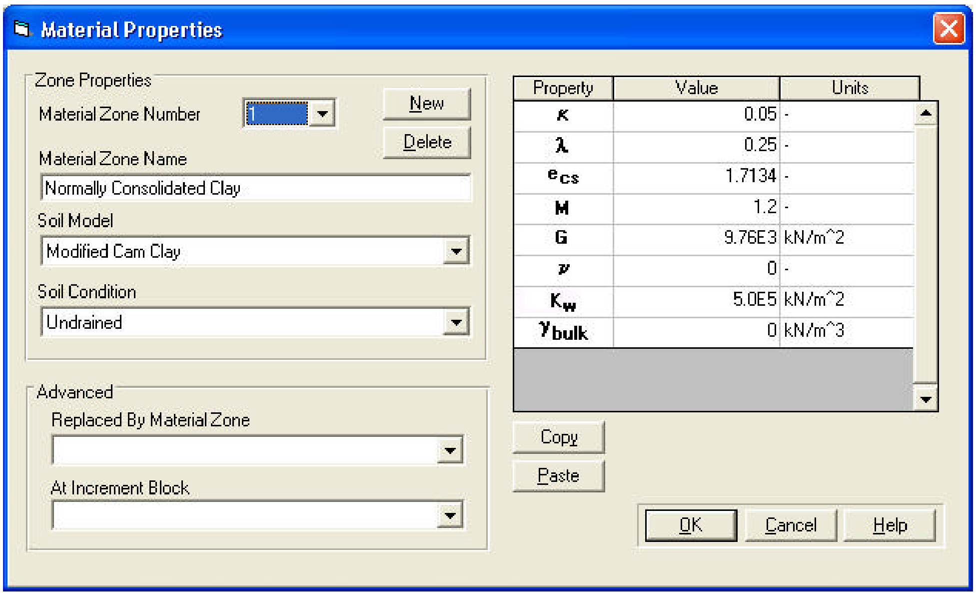


Figure 5 Material Properties (except J)

IMPORTANT NTOICE:
THE FIELD FOR PARAMETER J IS NOT YET PROGRAMMED IN THE CURRENT VERSION OF SAGE CRISP GUI (FEB 2002). THE USER MUST EDIT THE MPD FILE TO ENTER THE PARAMETER J. READ THE SECTION RUNNING ANALYSIS BELOW

Stress-controlled loading

With this test we apply a pressure to the top edge which is reversed (ie removed) for each cycle. A number of load blocks were created in SAGE CRISP. Each load block represents half a cycle (ie either loading or unloading). Each cycle would have a number of increments. It is also advised to use the Define option and vary the incremental load fractions in the load options menu as follows:

- Each Halves for the loading part of the cycle, ie increments reducing in size towards the end of the load block when stress path approaches the yield surface.
- Each Doubles for the unloading part of the cycle ie increments increasing in size as stress path moves away from yield surface into the elastic zone

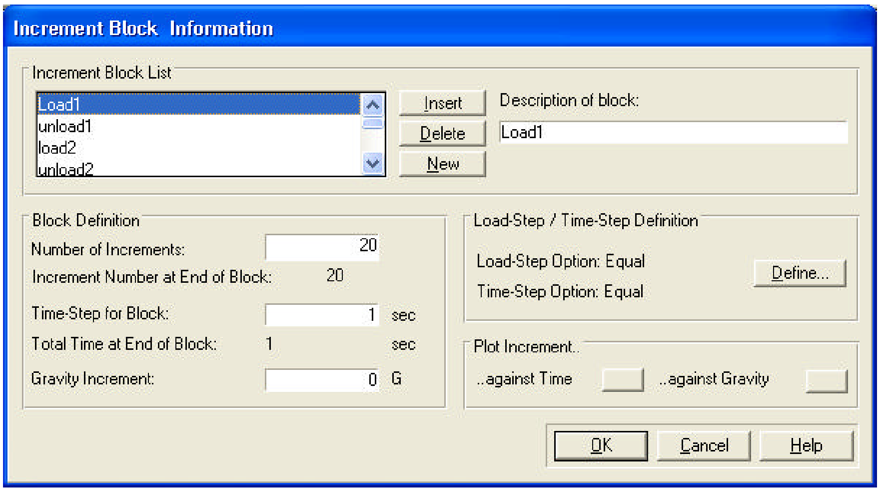


Figure 6 Load Blocks

The first load block is then selected an the top edge is selected. A pressure load q_c is then applied. This corresponds to

$$\frac{q_c}{2C_{uo}} = 0.75$$

where $C_{uo} = \frac{M}{4} p'_{co} \left(\frac{2p'_o}{p'_{co}} \right)^{k/l}$

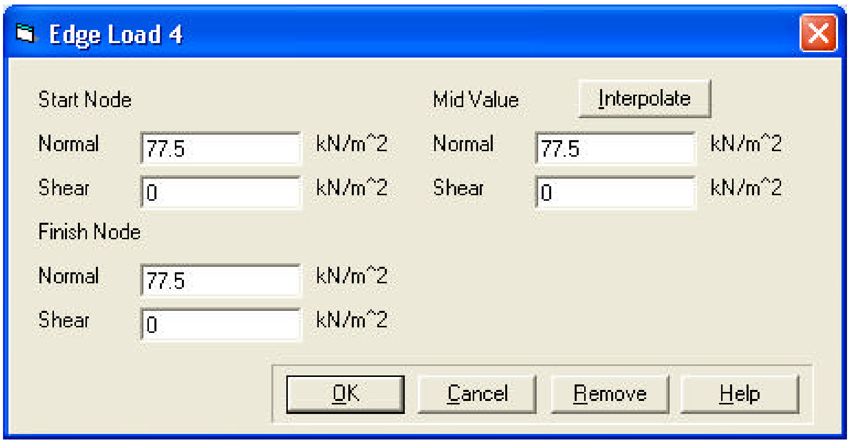


Figure 7 Applied stress for a loading case

The process is repeated for the next load block, but this time with a pressure of $-q_c$, ie unloading.

IMPORTANT NTOICE WHEN RUNNING THE ANALYSIS:
THE FIELD FOR PARAMETER J IN THE MATERIAL PROPERTIES IS NOT YET PROGRAMMED IN THE CURRENT VERSION OF SAGE CRISP GUI (FEB 2002). THE USER IS ADVISED TO DO THE FOLLOWING IN ORDER TO ACCOUNT FOR J:

- Create the mesh and enter all properties, loading etc.
- Click on File>Run Analysis
- Click on Create CRISP Files
- Go to Windows Explorer, or My Computer, locate the MPD file for this analysis and edit it using a text editor
- Locate Record D (the material properties) and enter the value for J in the 11th field of the material properties.
- Save the MPD file
- Go back to SAGE CRISP and click on Run Analysis Now. Do not click on Create CRISP files again as this will overwrite the MPD file you have just changed.

The following graphs are the results of the stress controlled cyclic test;

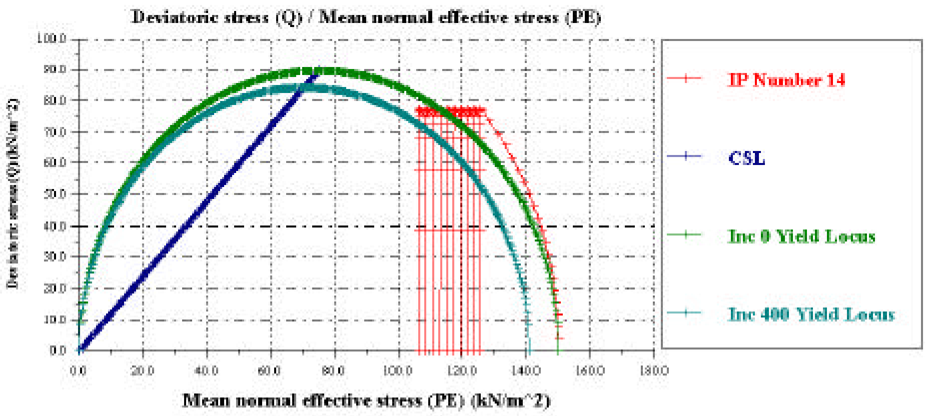


Figure 8 Variation of q v P' for stress controlled one-way cyclic loading of undrained normally consolidated problem showing initial and final yield surfaces.

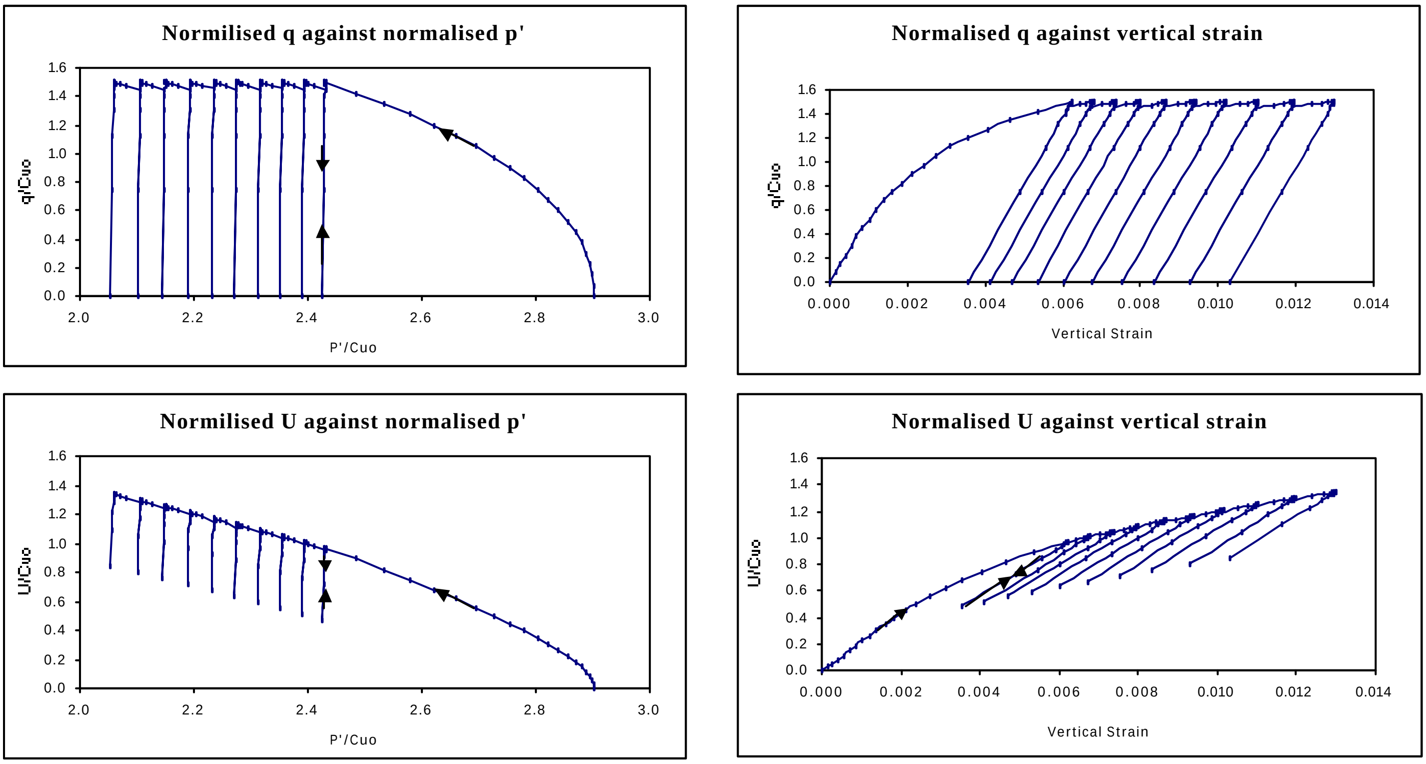


Figure 9 Results of one way stress controlled, undrained triaxial test with OCR=1, J =0.1

Strain Controlled Loading

The same FE model above was used with a loading of axial strain. Similar cell pressure was used as above. The axial strain loading represent a two way test in which the sample was subjected to compression as well as tension.

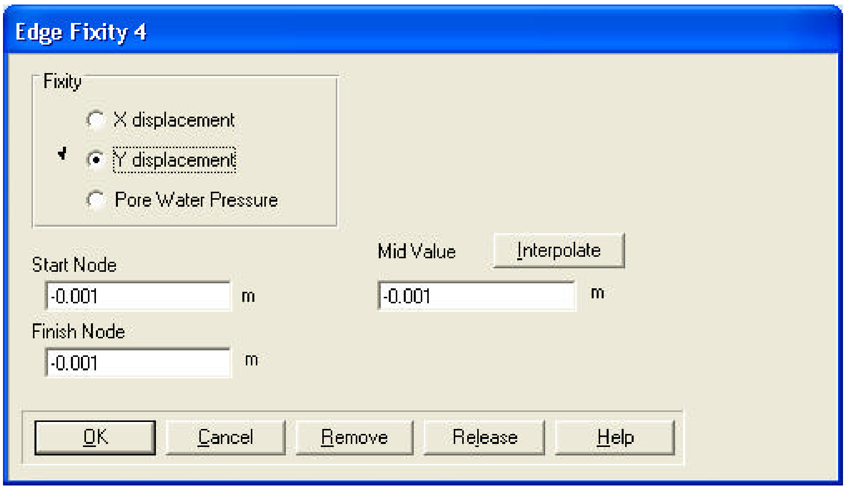


Figure 10 Input for edge fixities corresponding to a compressive axial strain of –0.001

The incremental fractions were again adjusted so that smaller increment are used when the stress path is approaching the yield surface and larger increments are used when the stress path is going away from the yield surface. The degradation parameter J was set to 0.1, and the material was assumed to be isotropic (k_o=1.0). Only 10 cycles were used.

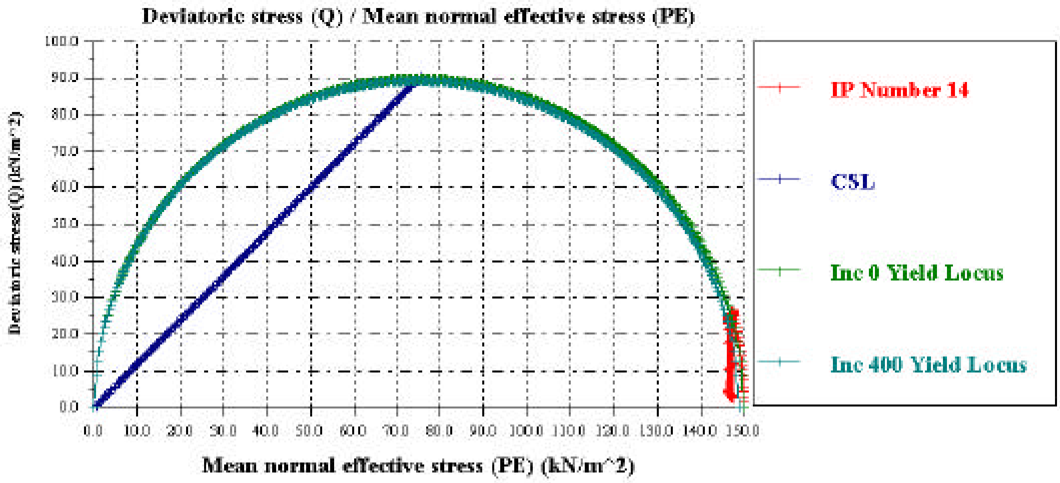


Figure 11 Stress path for two way axial strain test

In this type of strain controlled test the stress path moves steadily towards the critical state condition, oscillating between compression and tension, with the mean effective stress gradually reducing to zero. This behaviour implies that the soil is liquefying.

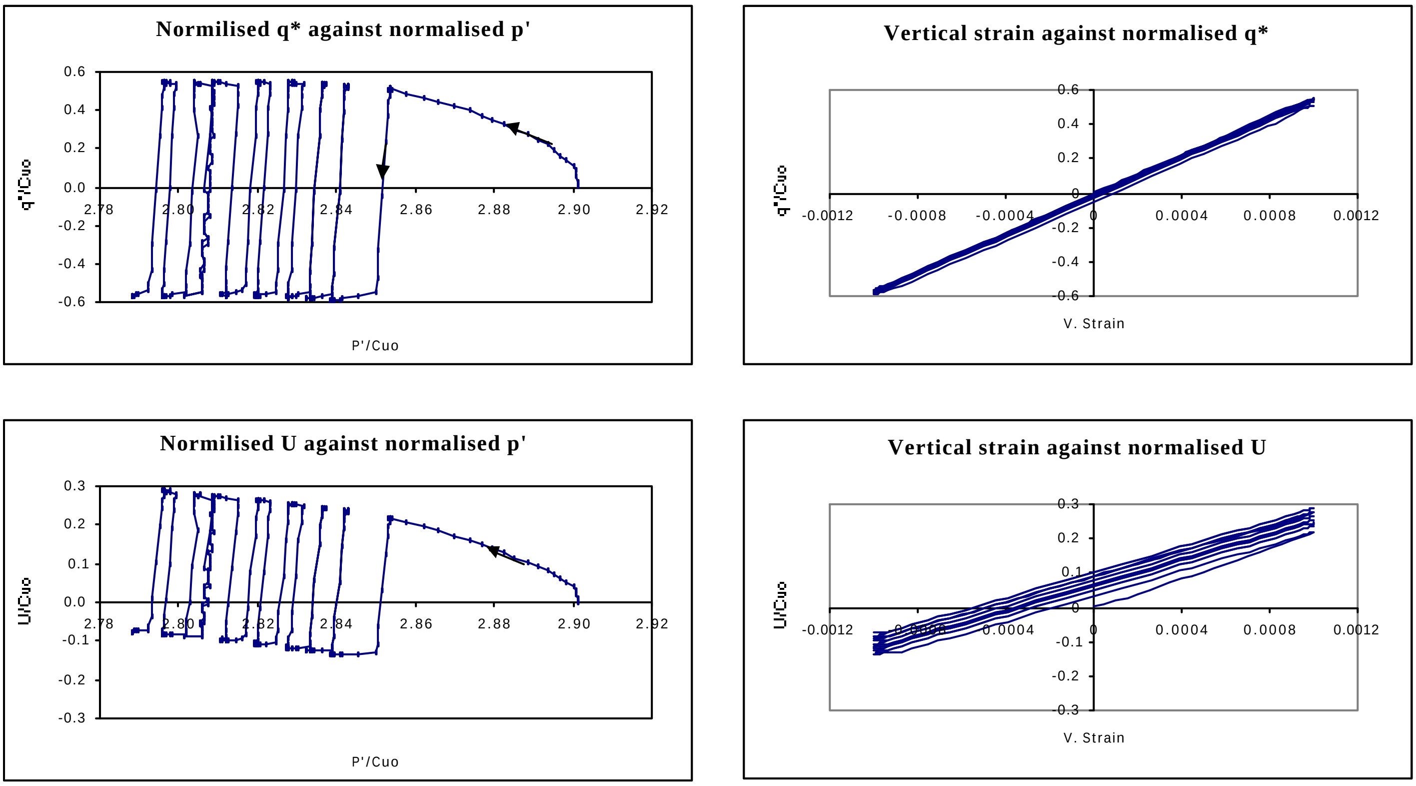


Figure 12 Results for two way axial strain using OCR=1, J =0.1 and G=200Cuo.$q^{*} = (s_1 - s_2)$

It can be seen from the graphs above that a gradual increase in pore pressure causes a reduction of effective mean stress as the soil slowly moves to liquefaction state.

Experimental determination of the model parameters

The parameters l and k can be obtained from oedometer or drained triaxial tests. The frictional parameter M is directly related to f which may also be obtained from drained triaxial tests. The elastic parameter G can either be obtained from bender element tests or it can be estimated as one third of the gradient of the deviator stress-axial strain curve on an unloading portion of an undrained triaxial test. It is also possible to us the effective Poisson's ratio n' instead of using G. Refer to SAGE CRISP Technical reference manual for further details.

It is possible to obtain a value for the parameter J from the results of one unloading-reloading cycle in a consolidation test. However, better estimates can be made using large number of cycles for undrained cyclic tests. Knowing the ratio $\frac{q_c}{2C_{uo}}$, and the number of cycles for a particular soil, it is possible to obtain J from the chart obtained for that type of soil. See figure 9.21 of the reference of Carter, Booker and Wroth

The reference above also provides other alternatives on estimating J (eg from the number of cycles required to generate a given excess pore pressure).

Comparison of FE results with experimental results.

The normalised effective mean stress decreases as the number of load cycles increases until the effective mean stress reaches zero (ie liquefaction state). Experiments have been done by Taylor and Bacchus in which sinusoidal strain-controlled cycles were applied to artificially prepared saturated clay samples. Comparison has been made in the reference by Carter, Booker and Wroth.

Conclusions

A soil model capable of producing the response of clay when subjected to repeated loading has been implemented into CRISP. The new model is based on the Modified Cam Clay model but with the addition of a degradation parameter to account for reduction in size of yield surface during cyclic loading. This new parameter may be determined from undrained cyclic triaxial tests.

The model is capable to produce many of the trends that have been observed in laboratory tests.

The model has the following limitations:

- A large number of increments is needed in order to produce an accurate stress path. The increments must be sufficiently small when the stress path is on or near the yield surface.
- The model cannot produce kinematic hardening in which the yield surface "shifts" during cyclic loading.

References:

1. Carter, J.P, Booker, J.R., Wroth, C.P. "A Critical State Soil Model for Cyclic Loading" published in *Soil Mechanics – Transient and Cyclic Loads*, 1982, John Wiley & Sons Ltd
2. Talor, P.W., and Bacchus, D.R., "Dynamic cyclic strain tests on a clay", *Proc. 7th Int. Conf. Soil Mechs. Found. Engng.*, Mexico, 1 401-409, 1969
3. SAGE CRISP Technical Reference Manual for version 4, by Rick Woods and Amir Rahim. The CRISP Consortium Ltd, 2002.